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Chiral symmetry breaking in dense matter

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Abstract. The quark condensate in nuclear matter is studied. We relate it to the nuclear sigma commutator, and then treat it as sigma commutator for quasi- particles in a self-consistent way. We find that the deviation from the linear expression is large at high density.

1 Introduction

Chiral symmetry breaking is relevant to a broad class of problems. The measure of the chiral symmetry is the value of the quark condensate $\langle\bar{q}q\rangle$. Cohen et al. [1] and Lutz et al. [2] have shown that this quantity can be linked to the free πN sigma term Σ_N through the model-independent relation

$$\frac{\langle\bar{q}q(\rho)\rangle}{\langle\bar{q}q(0)\rangle} = 1 - \frac{\Sigma_N \rho}{f_\pi^2 m_\pi^2} \tag{1}$$

which is valid at low density, where $f_\pi = 93MeV$ is pion decay constant and Σ_N=45 MeV [3]. Although (1) is only justified at low density, it is interesting to evalute this expression at nuclear matter saturation density ρ_o. If we take $\rho_o = 0.173/fm^3$, the extrapolation of the model independent result suggests that the in-medium quark condensate is roughly 25-50% smaller than the vacuum value and the chiral symmetry is restorted at high, but finite, nuclear matter density.

Chanfray and Ericson [4] have shown that in an exact treatment of this evolution the free commutator Σ_N has to be replaced by a nuclear sigma commutator per nucleon. Consider a piece of homogeneous nuclear matter of volume V, containing A nucleons with uniform density ρ. The quark condensate density can be taken as constant, thus we have

$$\Sigma_A = 2Vm_q[\langle\bar{q}q(\rho)\rangle - \langle\bar{q}q(0)\rangle] \tag{2}$$

and the density evolution

$$\frac{\langle\bar{q}q(\rho)\rangle}{\langle\bar{q}q(0)\rangle} = 1 - \frac{\Sigma_A \rho}{A f_\pi^2 m_\pi^2} \tag{3}$$

If we introduce the effective Σ-commutator for a nucleon imbedded in the nuclear medium $\tilde{\Sigma}_N = \Sigma_A/A$. (3) can be rewritten as

$$\frac{\langle\bar{q}q(\rho)\rangle}{\langle\bar{q}q(0)\rangle} = 1 - \frac{\tilde{\Sigma}_N \rho}{f_\pi^2 m_\pi^2} \tag{4}$$

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Ericson [5] linked the quark condensate with the nuclear sigma term and treated it as a scattering problem of soft pions on the nucleus described as a collection of interacting nucleons. This produces a reaction of the nuclear medium against the restoration of chiral symmetry. They find $\langle\bar{q}q(0)\rangle$ goes to zero only for infinite densities. But Birse and McGovern [6] have a different viewpoint on this problem and point out that the scattering term should not be treated separately from other terms of similar magnitude. Thus it is worth investigating further. In this note we try to consider the problem in an other way, we would like to have a discussion on the relation between hadron mass and the chiral symmetry restoration.

2 The mass reduction and the chiral symmetry

In recent years there are many papers appearing in the literatures concerning the influence of the effective mass. The property of vacuum is tightly connected to the quark condensate. The change of quark condensate reflects the change of vacuum and a direct consequence is the variation of the hadron mass. It is evident in bag model. G.E.Brown et al. [7–10] proposed that all mass scales in-medium are changed with density in the same manner and mass of hadron with chiral quark scales as $\langle\bar{q}q\rangle_\rho^{1/3}$. If one defines the effective mass to be the Lorentz-scalar part of the effective nucleon self-energy, then recent QCD sum rule calculations suggest that the effective nucleon mass scales as $\langle\bar{q}q\rangle_\rho$ [11]. On the other hand, the Σ commutator for a free nucleon can be related to the isospin symmetric scattering length $a_N^+(0)$ [12,5] by

$$a_N^+(0) = -\frac{\Sigma_N}{4\pi f_\pi^2} \tag{5}$$

and

$$a_N^+(0) \propto -\frac{m_\pi M_N}{f_\pi^2(M_N + m_\pi)} \tag{6}$$

The Σ commutator for a nucleon in medium can be viewed as the Σ commutator for a quasi-particle with effective mass M_N^* and m_π^* instead of M_N and m_π for free particles. Thus we have

$$\tilde{\Sigma}_N = \Sigma_N \frac{m_\pi^* M_N^* / (m_\pi^* + M_N^*)}{m_\pi M_N / (m_\pi + M_N)} \tag{7}$$

Now we modify the approach given by [4], which gives (4). We would like to take into account the influence of the medium on the quark condensate in a self-consistent way, including the influence of the quark condensate on the Σ-commutator in turn, thus the effective Σ-commutator in (4) should be expressed as in (7). We keep $f_\pi^2 m_\pi^2$ unchanged, as it is connected to the factor $\langle \bar{q}q(0)\rangle$ directly. There are many different ways to calculat the effective mass, here we only use the mass scaling, as it is widely accepted and used to deal with the properties of hadron at high density nuclear matter. If the effective mass of hadron with chiral quarks scales as $\langle \bar{q}q\rangle_\rho$ [11], (7) reduces to $\tilde{\Sigma}_N = \Sigma_N \langle \bar{q}q(\rho)\rangle / \langle \bar{q}q(0)\rangle$. Thus a self-consistent treatment gives

$$\frac{\langle \bar{q}q(\rho)\rangle}{\langle \bar{q}q(0)\rangle} = 1 - \frac{\Sigma_N \rho}{f_\pi^2 m_\pi^2} \frac{\langle \bar{q}q(\rho)\rangle}{\langle \bar{q}q(0)\rangle} \tag{8}$$

The variation of the effective Σ-commutator gives influence on the quark condensate and in turn the change of quark condensate will influence the effective Σ. Such a relation is shown in (8). (8) can be rewritten as

$$\frac{\langle \bar{q}q(\rho)\rangle}{\langle \bar{q}q(0)\rangle} = \frac{1}{1 + \frac{\Sigma_N \rho}{f_\pi^2 m_\pi^2}} \tag{9}$$

That is just what obtained by Ericson [5]. Ericson treated the problem of the nuclear sigma term as a scattering problem, but taking into account the distortion of the pion wave or equivalently the coherent rescattering of pion. Our result implies that the decreasing of effective mass of hadron at high density may reduce the restoration rate and cause the large deviation from the linear extrapolation of (1), even if the rescattering is not considered.

We calculate $\frac{\langle \bar{q}q(\rho)\rangle}{\langle \bar{q}q(0)\rangle}$ with mass scaling as $\langle \bar{q}q\rangle_\rho^{1/3}$ also, and show the results in Fig. 2. We find that in these two cases, $\langle \bar{q}q(\rho)\rangle \to 0$ only at $\rho \to \infty$. Based on the bag model and linear σ model we obtained that the mass of hadron with chiral quarks scales as $\langle \bar{q}q\rangle_\rho^{1/4}$ [13,14]. The results from this scaling are shown and compared with others in Fig. 1 and Fig. 2. We find that the deviation of the effective mass is not small even at low densities, but the quark condensate goes to zero only for infinite densities in the approaches of different mass scales.

In the above discussions the rescattering correction is not considered. If we include these contributions, not only the contribution from the distortion effects as given by Ericson, but also the contributions of the similar magnitude from other processes as given by Birse and McGovern, we have

$$\tilde{\Sigma}_N = \Sigma_N + \frac{3}{2} \frac{\Sigma_N^2}{f_\pi^2 m_\pi^2} \rho \tag{10}$$

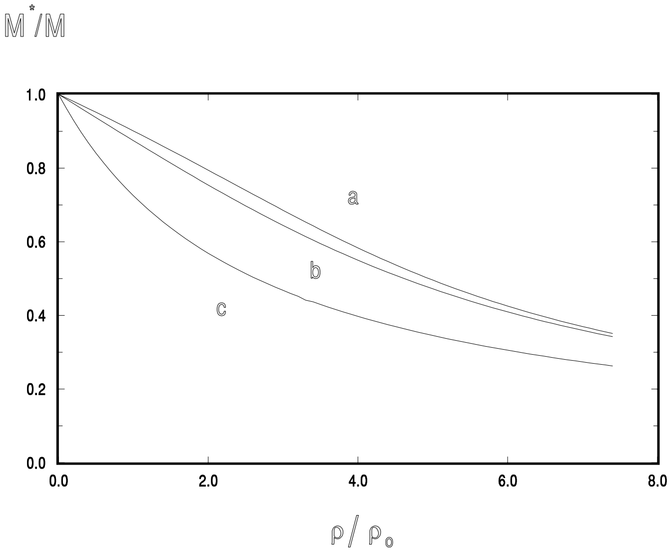

Fig. 1. Evolution of effective nucleon mass M^*/M as a function of the density ρ/ρ_o, where $\rho_o = 0.173/fm^3$ is the nuclear matter density. Mass scales as a $\langle \bar{q}q\rangle_{\rho_N}^{1/4}$, b $\langle \bar{q}q\rangle_{\rho_N}^{1/3}$, c $\langle \bar{q}q\rangle_{\rho_N}$ and calculated with $\Sigma_N = 45MeV$, $f_\pi = 93MeV$

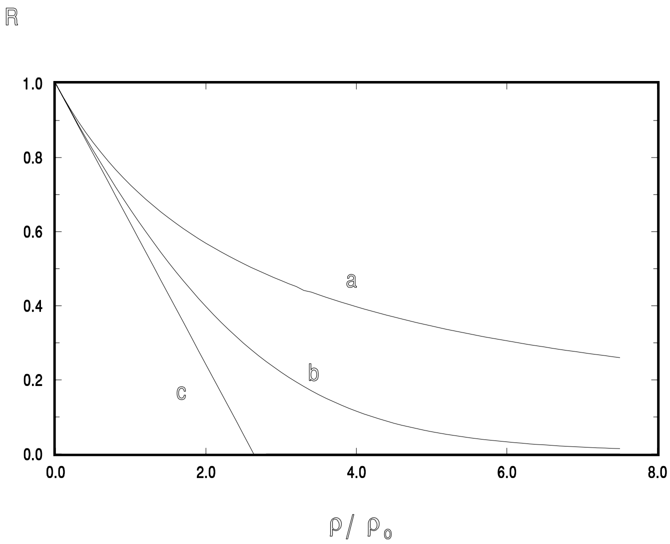

Fig. 2. Evolution of the quark condensate $R = \langle \bar{q}q(\rho)\rangle / \langle \bar{q}q(0)\rangle$ is a function of the density ρ/ρ_o, where $\rho_o = 0.173/fm^3$ is the nuclear matter density. Mass scales as a $\langle \bar{q}q\rangle_{\rho_N}$, b $\langle \bar{q}q\rangle_{\rho_N}^{1/4}$. c is the extrapolation of (1)

where the reduction of effective mass is not cosidered. If the effective mass scales as $\langle \bar{q}q\rangle_\rho$ the Σ_N term in the right hand side should be replaced by

$$\Sigma_N \frac{\langle \bar{q}q(\rho)\rangle}{\langle \bar{q}q(0)\rangle}$$

thus we have

$$\frac{\langle \bar{q}q(\rho)\rangle}{\langle \bar{q}q(0)\rangle} = 1 - \frac{\Sigma_N \rho}{f_\pi^2 m_\pi^2} \frac{\langle \bar{q}q(\rho)\rangle}{\langle \bar{q}q(0)\rangle} - \frac{3}{2} \frac{\Sigma_N^2 \rho^2}{f_\pi^4 m_\pi^4} \frac{\langle \bar{q}q(\rho)\rangle^2}{\langle \bar{q}q(0)\rangle^2} \tag{11}$$

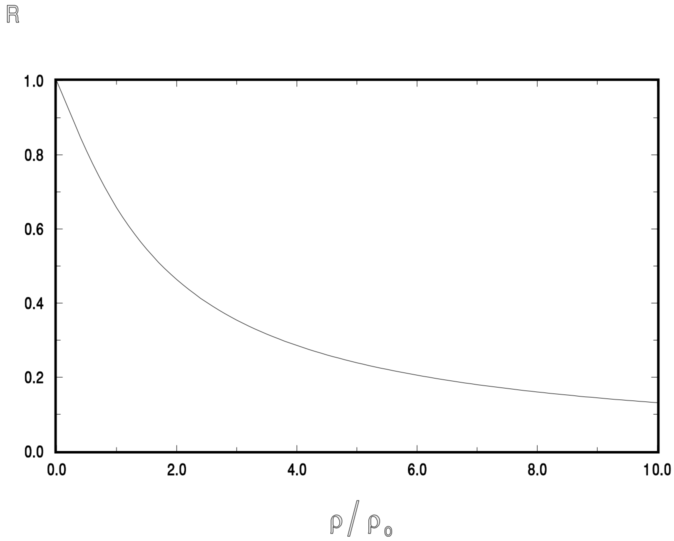

Fig. 3. Evolution of the quark condensate given by (11) $R = \langle\bar{q}q(\rho)\rangle \,/\, \langle\bar{q}q(0)\rangle$ is a function of the density ρ/ρ_o, where $\rho_o = 0.173/fm^3$ is the nuclear matter density and mass scales as $\langle\bar{q}q\rangle_{\rho_N}$

We show the numerical results in Fig. 3. The large deviation from the linear extraporation is seen evidently. The enhancement of chiral symmetry restoration in the sigma model given by [6] is greatly reduced.

3 Summary and discussions

(1) We have studied the evolution with the density of the quark condensate and related it to the nucleon sigma commutator. The variation of the vacuum will in turn influence the hadron mass. Thus we use the effective mass in-medium to treat the scattering length. The approach of mass scales is used here and the self-consistent treatment gives the evolution with the density of the quark condensate.

(2) Birse and McGovern [6] point out that some terms should be added to Ericson's calculation, and these terms will enhance the chiral symmetry restoration. Here the Ericson's result is reproduced, but in a different way. We show the mass dropping may reduce the restoration rate. Even if the rescattering correction is considered as Birse and McGovern we find the enchancement is greatly reduced.

(3) In the approach of bag model the mass of pion scales as other hadrons with light quarks obviously. In [15], we show that the soft pion enhancement observed by NA35 Collabration can be understood as a consequence of the pion mass dropping. That is why we take the pion mass scales as other hadrons here.

(4) In the region $\rho < \rho_o$ the difference of quark condensate among these models is not large, but the difference grows with the increasing of the nucleon density. If we extraporate (1), which is justified valid at low density, to high density, we can find the value of the chiral restoration density, $\rho_c = 2.75\rho_o$, but we cannot find the finite value of the chiral restoration density in the scaling approach. The hadron mass in medium at saturation density is roughly 10% smaller than that in the vacuum, if the mass scales as $\langle\bar{q}q\rangle_\rho^{1/4}$ or $\langle\bar{q}q\rangle_\rho^{1/3}$. If the mass scales as $\langle\bar{q}q\rangle_\rho$, we find the decrease is about 27%.

(5) The chiral symmetry breaking in dense matter is a widely concerned problem. It is well known that (1) is valid at low density. The direct extrapolation shows that the chiral symmetry is restorted at high, but finite, nuclear matter density. This conclusion is supported by some model dependent results [1,2,6]. On the other hand, some models show that the chiral symmetry can only be restorted at infinite density or the breaking may even be enchanced by the nuclear matter. [5,16] Thus a further investigation is necessary. Here we proposed that (1) should be modified and the mass scaling, which is widely accepted and "model independent", should be taken into account to make (1) self-consistent. The results from the modified equation are different from those of the direct extrapolation. We find that the chiral symmetry is only restorted for the infinite nucleon density. As our arguments are based on an essential constraint, they are more convincible than those from special models. Our discussion is valid at these densities where the medium can be taken as a collection of nucleons.

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